Chap 2-5: Motion at Constant Acceleration

Notation:

Initial time	(t_1) is zero:	$t_0=0$
final time	(t_2) is t :	t
initial position	(x_1) is :	x_0
final position	(x_2) is :	x
initial velocity	(v_1) is :	v_0
final velocity	(v_2) is :	v

Average Velocity: $\bar{v} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$ Since $t_0=0$.

Acceleration: $a = \frac{v - v_0}{t}$ Assumed to be a constant.

Chap 2-5: Motion at Constant Acceleration

Generic Problem: Given an initial velocity, acceleration and time, determine the final velocity.

$$a = \frac{v - v_0}{t} \Rightarrow \boxed{v = v_0 + at}$$

Example: A rocket's acceleration is $50 m/s^2$, how fast will it be going after 10s if it starts at rest?

Given:

$a = 50\ m/s^2$

$v_0 = 0$

$t = 10\ s$

$$\boxed{v = 0 + (50\ m/s^2)\ (10\ s) = 500\ m/s}$$

Chap 2-5: Motion at Constant Acceleration

Generic Problem: Find the position of an object that is undergoing a constant acceleration.

$$\bar{v} = \frac{x - x_0}{t} \Rightarrow \boxed{x = x_0 + \bar{v}t}$$

Also, at constant acceleration we have that:

$$\left.\begin{aligned} \bar{v} &= \frac{v_0 + v}{2} \\ v &= v_0 + at \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} x &= x_0 + \left(\frac{v_0 + v}{2}\right)t \\ x &= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t \\ &\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2} \end{aligned}\right.$$

Chap 2-5: Motion at Constant Acceleration

If time is unknown:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v_0 + v}{2}\right)t \quad \textbf{and} \quad v = v_0 + at \Rightarrow t = \frac{v - v_0}{a}$$

Substituting in

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - {v_0}^2}{2a}$$

$$v^2 = {v_0}^2 + 2a(x - x_0)$$

Chap 2-5: Motion at Constant Acceleration

Equations of Motion At Constant acceleration

$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{v} = \frac{v_0 + v}{2}$$

Chap 2-5: Motion at Constant Acceleration

Example: For a car to start from rest and reach speed of 30 m/s (≈ 65 mile/hour) in 6 s with constant acceleration,

A) What must its acceleration be?

B) How far did it travel after 6 s?

Given:

- x_0 = 0 m
- v_0 = 0 m/s
- v = 30 m/s
- t = 6s

A)
$$a = \frac{v - v_0}{t} = \frac{30\frac{m}{s}}{6s} \Rightarrow a = 5\frac{m}{s^2}$$

B)
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = \tfrac{1}{2}5\frac{m}{s^2}(6s)^2$$

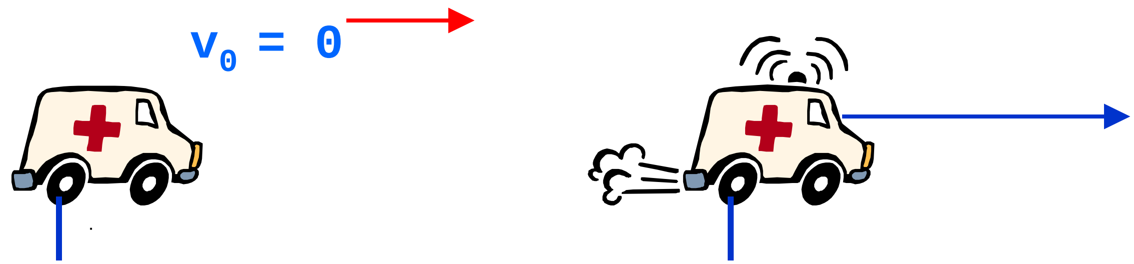
$$x = 90m$$

Chap 2-6: Problem Solving:

A car starts at rest, then it accelerates at a constant 3 m/s^2, how long will it take the car to go 50 m?

Strategy:

1) Draw a diagram:



2) Make a table:

Known	Wanted
$x_0 = 0$	t
$x = 50$ m	
$v_0 = 0$	
$a = 3$ m/s^2	

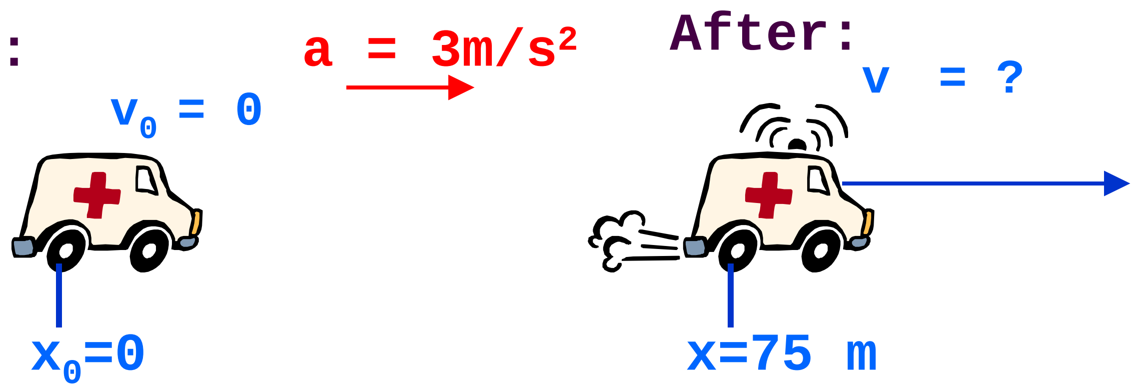
3) Solve for the unknown:

$$x = \tfrac{1}{2}at^2 \Rightarrow t^2 = \frac{2x}{a}$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(50m)}{3m/s^2}} = 5.77s$$

Chap 2-6: Problem Solving:

A car starts at rest, then it accelerates at a constant 3 m/s², how fast will it go after 75 m?



Strategy:

1) Draw a diagram:

2) Make a table:

Known	Wanted
$x_0 = 0$	v
x = 75 m	
$v_0 = 0$	
$a = 3\ m/s^2$	

3) Solve for the unknown:

$$v^2 = 2ax$$

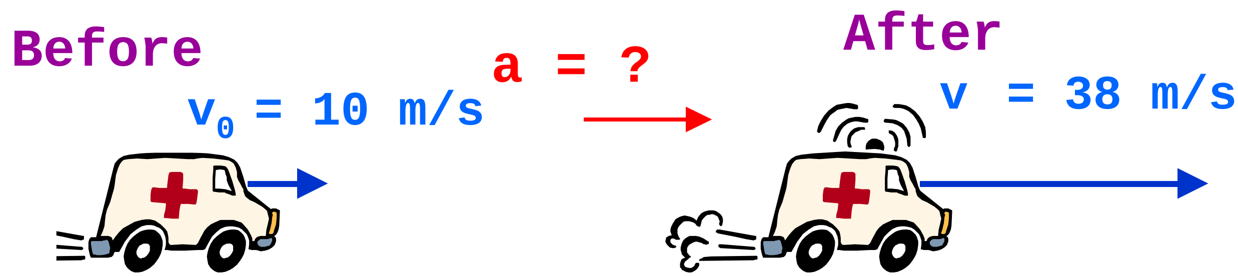
$$v = \sqrt{2ax} = \sqrt{2(3\frac{m}{s^2})(75m)} = 21.2\,\frac{m}{s}$$

Chap 2-6: Problem Solving:

A car is going at 10 m/s, 7 s later it is going at 38 m/s, find the acceleration (assume it is constant).

Strategy:

1) Draw a diagram:



2) Make a table:

Known	Wanted
$x_0 = 0$	a
$v_0 = 10$ m/s	
v = 38 m/s	
t = 7 s	

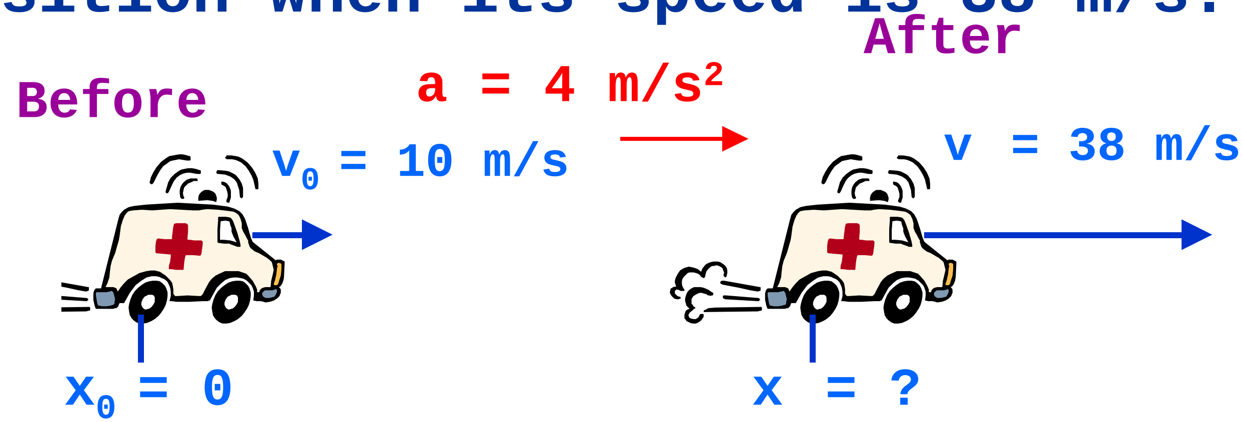
3) Solve for the unknown:

$$v = v_o + at$$

$$a = \frac{v - v_o}{t} = \frac{28\frac{m}{s}}{7s} = 4\frac{m}{s^2}$$

Chap 2-6: Problem Solving:

A car's speed is 10 m/s and its acceleration is 4 m/s^2, find its position when its speed is 38 m/s.



Strategy:

1) Draw a diagram:

2) Make a table:

Known	Wanted
$x_0 = 0$	x
$v_0 = 10$ m/s	
$v = 38$ m/s	
$a = 4$ m/s^2	

3) Solve for the unknown:

$$v^2 = v_o^{\ 2} + 2a(x - x_0)$$

$$x = \frac{v^2 - v_0^{\ 2}}{2a} = \frac{(38\frac{m}{s})^2 - (10\frac{m}{s})^2}{2(4\frac{m}{s^2})} = 168m$$

Chap 2-7: Falling Objects:

An object falling freely near the Earth's surface is an example of a uniformly accelerated motion.

In the absence of air resistance, **ALL** objects fall with the **same** constant acceleration.

This acceleration is due to gravity, it is denoted by g:

$$g = 9.80 \text{ m/s}^2$$

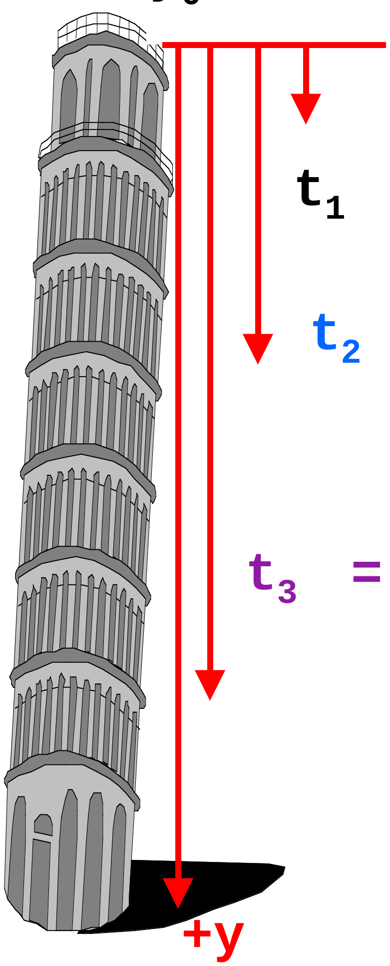
We can rewrite the equation of motion, substituting g for a and y for x (since it is a vertical motion):

$$y = y_0 + v_0 t + \tfrac{1}{2} g t^2$$

Chap 2-7: Falling Objects:

Example: Drop a ball from a tower, find its position (Choose y to be positive in the downwards direction)

$y_0 = 0$



$t_1 = 1.00$ s

$t_2 = 2.00$ s

$t_3 = 3.00$ s

Remember $g = 9.80\ m/s^2$

If $v_0 = 0$ then:

$$y = y_0 + v_0 t + \tfrac{1}{2} g t^2$$

$$y = \tfrac{1}{2} g t^2$$

$y_1 = (9.80\ m/s^2)(1.00s)^2 / 2 = 4.90$ m

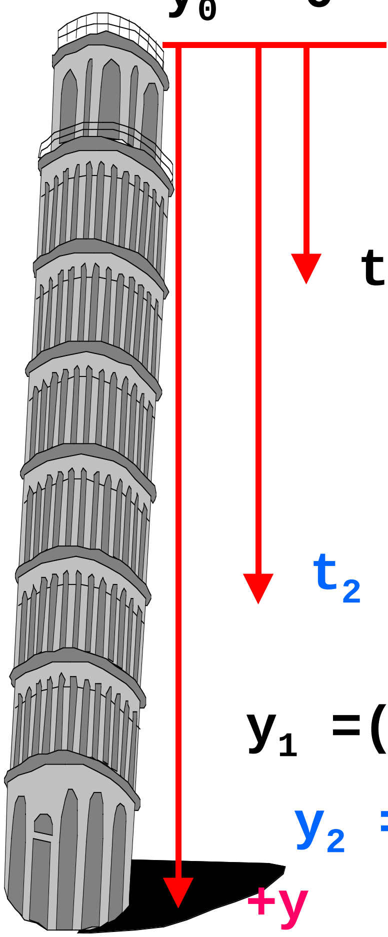
$y_2 = (9.80\ m/s^2)(2.00s)^2 / 2 = 19.6$ m

$y_3 = (9.80\ m/s^2)(3.00s)^2 / 2 = 44.1$ m

Chap 2-7: Falling Objects:

Example: Drop a ball from a tower, find its position

$y_0 = 0$



Remember $g = 9.80\ m/s^2$

$t_1 = 1.00$ s

If $v_0 = 10$ m/s then:

$$y = y_0 + v_0 t + \frac{1}{2} g t^2$$

$$y = v_0 t + \frac{1}{2} g t^2$$

$t_2 = 2.00$ s

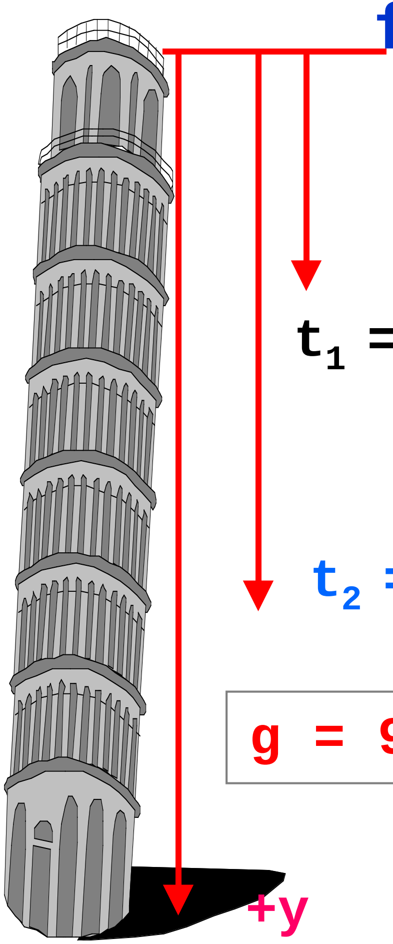
$y_1 = (10\ m/s)(1.00\ s) + (9.80\ m/s^2)(1.00 s)^2 / 2 = 14.9\ m$

$y_2 = (10\ m/s)(2.00\ s) + (9.80\ m/s^2)(2.00 s)^2 / 2 = 39.6\ m$

Chap 2-7: Falling Objects:

Example: Drop a ball from a tower, find its velocity

$y_0 = 0$



$t_1 = 1.00$ s

$t_2 = 2.00$ s

$g = 9.80$ m/s²

If $v_0 = 10$ m/s then:

$$v = v_0 + at$$

$v_1 = (10 \text{ m/s}) + (9.80 \text{ m/s}^2)(1.00\text{s}) = 19.8 \text{m/s}$

$v_2 = (10 \text{ m/s}) + (9.80 \text{ m/s}^2)(2.00\text{s}) = 29.6 \text{m/s}$

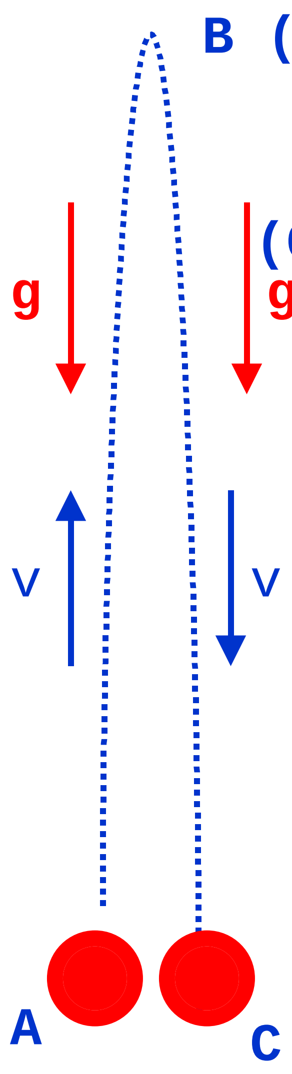
If $v_0 = 0$ then:

$$v = at$$

$v_1 = (9.80 \text{ m/s}^2)(2.00\text{s}) = 9.80 \text{ m/s}$

$v_2 = (9.80 \text{ m/s}^2)(2.00\text{s}) = 19.6 \text{ m/s}$

Chap 2-7: Falling Objects:



Throwing a ball upwards

$v_0 = 20$ m/s; $a = -g = -9.80$ m/s^2

(Choose y to be positive in the upwards direction)

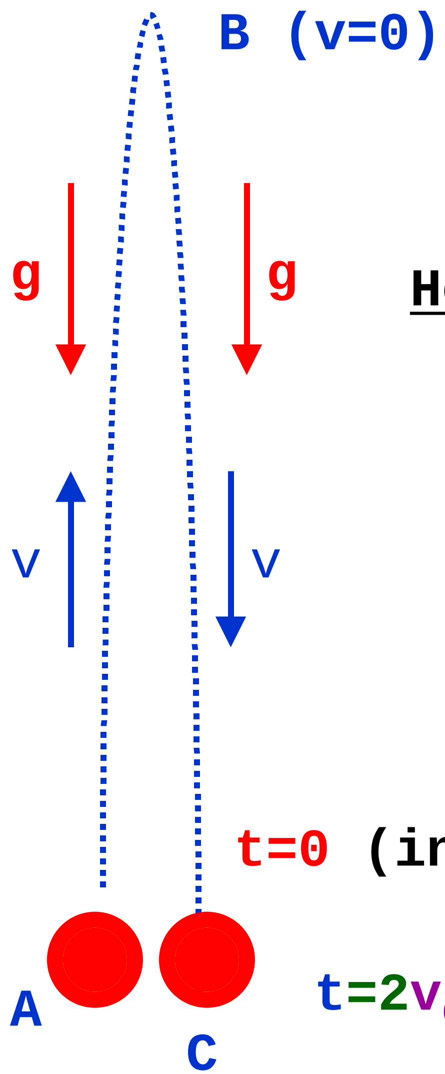
How high will it go (how high is Point B)?

$$v^2 = v_0^2 + 2ay$$

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - \left(20\frac{m}{s}\right)^2}{2\left(-9.80\frac{m}{s^2}\right)} = 20.4m$$

Chap 2-7: Falling Objects:

Throwing a ball upwards



v_0 = 20 m/s   a = –g = –9.80 m/s^2

(Choose y to be positive in the upwards direction)

How long will it be in the air?

$$y = v_0 t + \frac{1}{2} a t^2$$

Find solution for y = 0 (when it is caught)

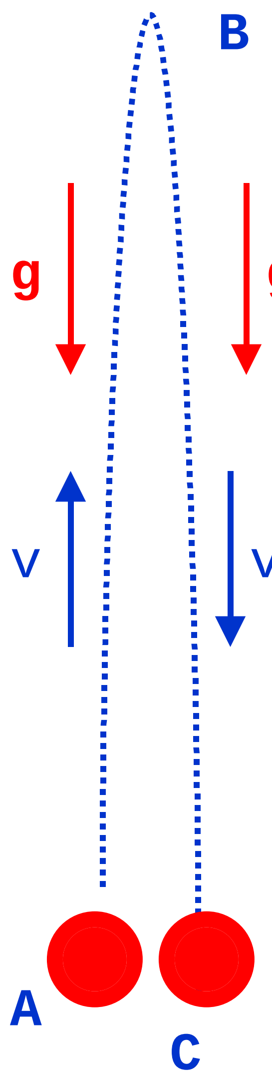
$0 = v_0\ t - g\ t^2 / 2 = (v_0 - g\ t / 2)\ t$

There are two solutions:

t=0 (initial point, before it was thrown, Point A)

$t = 2v_0/g = 2(20\ m/s)/(9.80\ m/s^2) = 4.08\ s$ (Point C)

Chap 2-7: Falling Objects:



Throwing a ball upwards

v_0 = 20 m/s; a = −g = −9.80 m/s^2

(Choose y to be positive in the upwards direction)

How long will it take the ball to get to point B?

v = 0

$v = v_0 + a\ t$ ⟹ $t = v_0 / g = (20\ m/s)/(9.80\ m/s^2)$

t = 2.04 s

Half the time it took to reach Point C

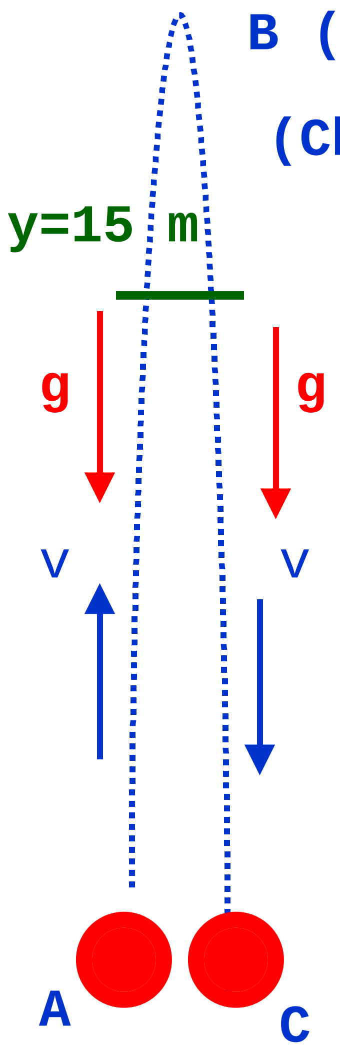
How fast will the ball go at point C ?

$v = v_0 + a\ t = (20\ m/s) - (9.80\ m/s^2)(4.08\ s) = -20 m/s$

Same magnitude, different sign of the initial velocity.

Chap 2-7: Falling Objects:

Throwing a ball upwards



v_0 = 20 m/s; a = −g = −9.80 m/s^2

(Choose y to be positive in the upwards direction)

At what time t will it reach a point y=15.0 m ?

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2 \Rightarrow \tfrac{1}{2} g t^2 - v_0 t + y = 0$$

Using the solution of the Quadratic Equation

$$t = \frac{v_0 \pm \sqrt{v_0^2 - 4gy/2}}{g}$$

$$t = \frac{20\frac{m}{s} \pm \sqrt{(20\frac{m}{s})^2 - 2(9.80\frac{m}{s^2})(15.0m)}}{9.80\frac{m}{s^2}}$$

t = 0.990 s   t = 3.09 s

Quadratic Equation

$$at^2 + bt + c = 0$$

Has the solution:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Edge on Galaxy NGC 4565

This galaxy is about 40 million light years away