

Kinematics in Two Dimensions; Vectors

Chap 3-1: Vectors and scalars

Vector: A quantity that has both magnitude and direction. For Example: Velocity, Acceleration, Force, Momentum etc.

We will denote vectors as bold face quantities

$\mathbf{D}, \mathbf{v}, \mathbf{F}, \mathbf{p}$

Scalar: A quantity that has only a magnitude.

For example: Temperature, Mass, Time etc.

We will denote scalars as regular face quantities

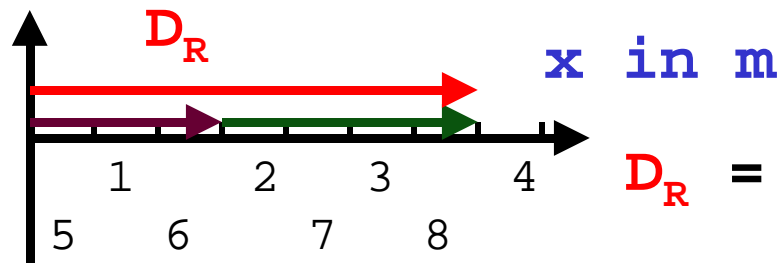
T, m, t

Chap 3-2: Graphical addition of vectors

Summing parallel vectors

Adding

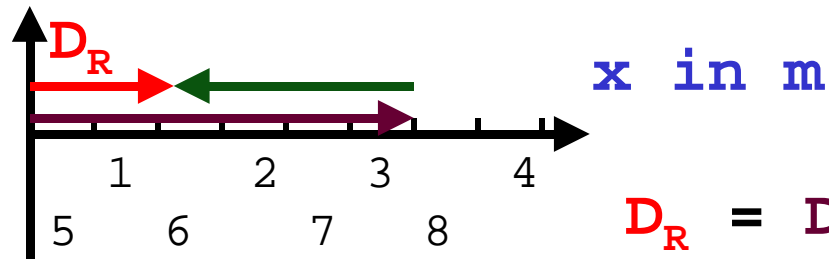
Displacement vector D_i



Resultant vector D_R

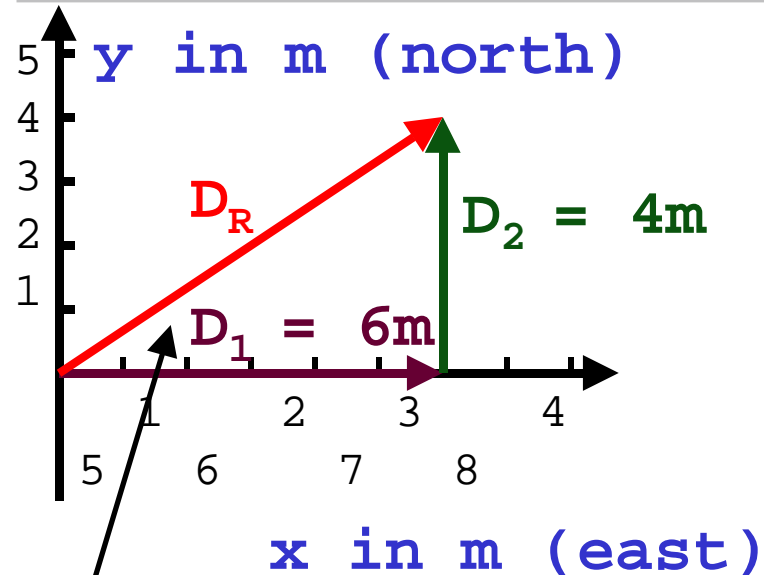
$$D_R = D_1 + D_2 = 3 \text{ m} + 4 \text{ m} = 7 \text{ m}$$

Subtracting



$$D_R = D_1 - D_2 = 6 \text{ m} - 4 \text{ m} = 2 \text{ m}$$

Chap 3-2: Graphical addition of vectors



$$\theta = \text{atan}(D_2 / D_1)$$

$$\theta = 34^\circ$$

$$D_R = D_1 + D_2$$

vector equation

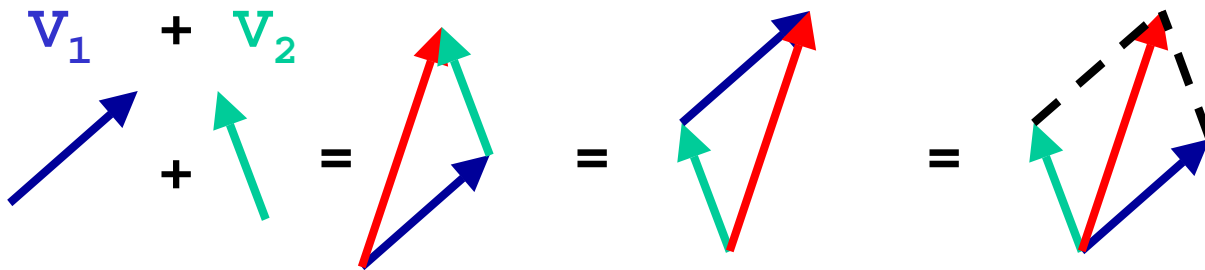
The magnitude of D_R is:

$$D_R = \sqrt{D_1^2 + D_2^2}$$

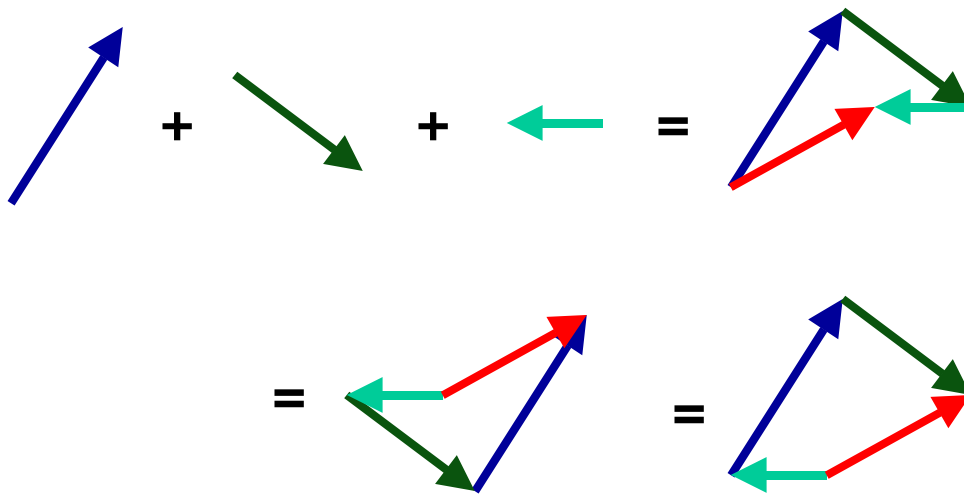
This is true only if the vectors are perpendicular to each other.

$$\text{So } D_R = 7.2 \text{ m}$$

Chap 3-2: Graphical addition of vectors

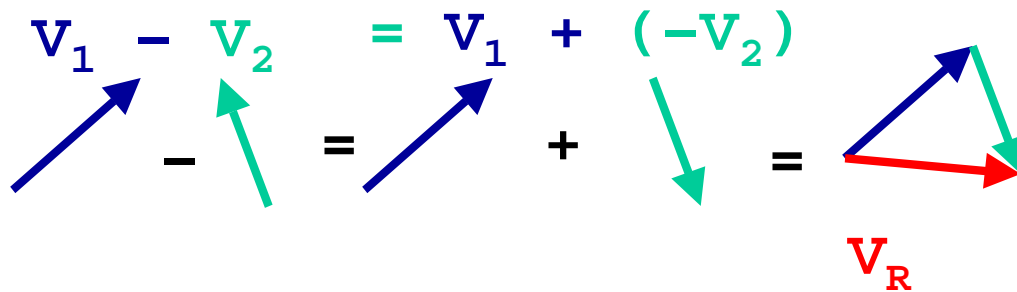


Draw V_1 to scale, draw V_2 to scale, and so on. Place the tail of the one on the tip of the other.



The resultant vector is the vector that goes from the tail of the first to the tip of the last.

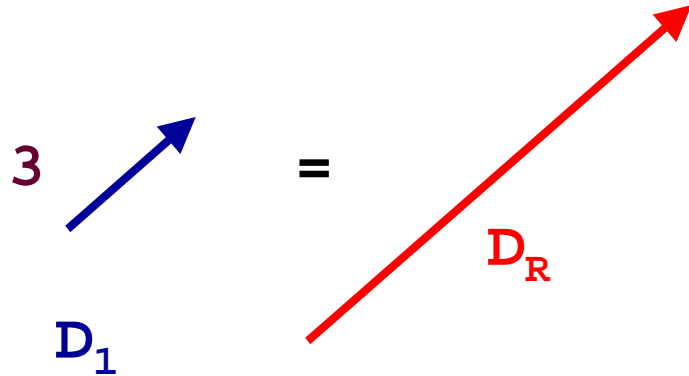
Chap 3-3: Graphical subtraction of vectors



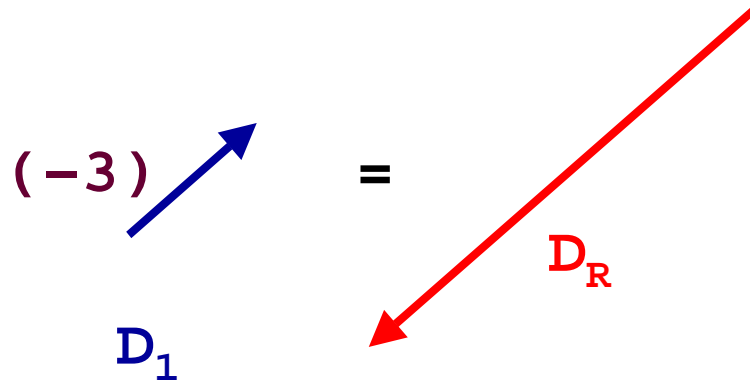
Draw V_1 to scale,
draw V_2 to scale.
Reverse the
direction of the
one to subtract.

The resultant
vector is the
vector that goes
from the tail of
the first to the
tip of the
reversed last.

Chap 3-3: Graphical multiplication of vectors by scalars



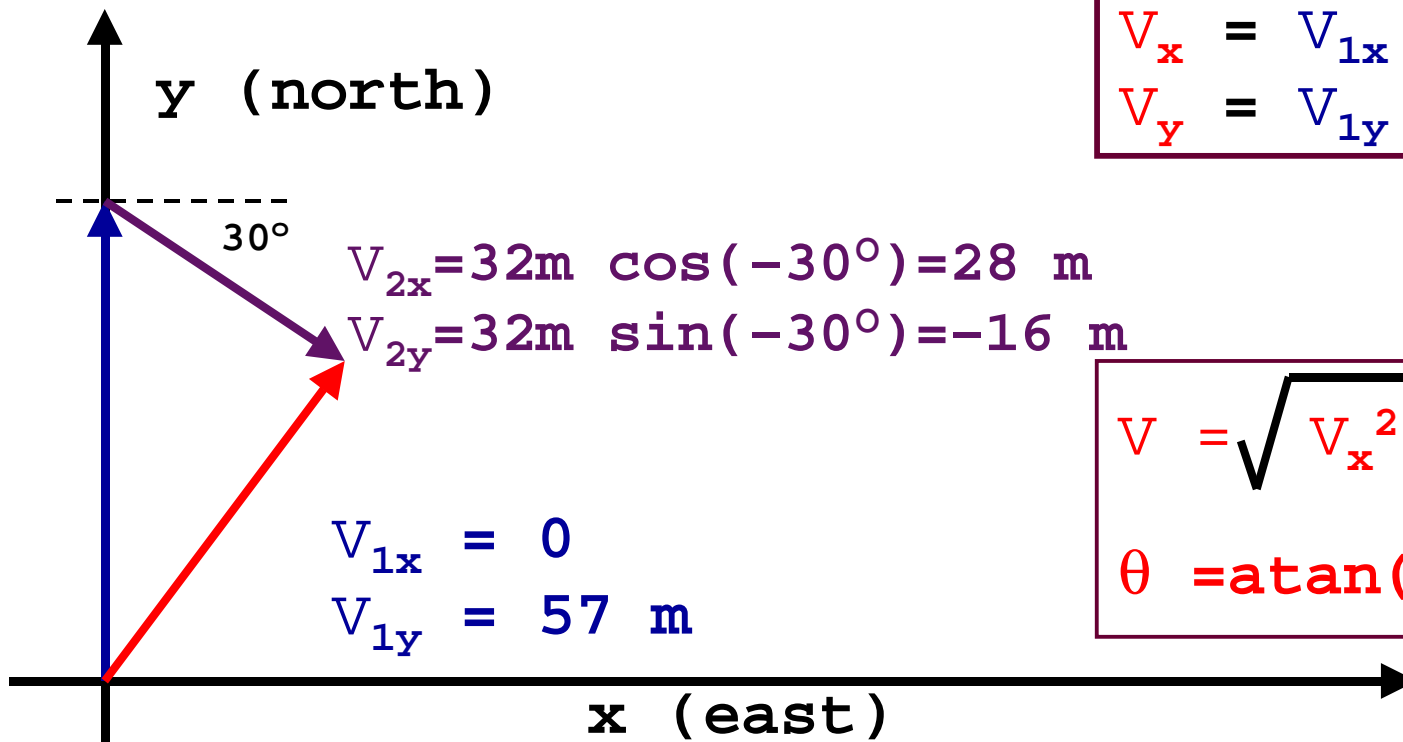
Draw D_1 to scale, multiply the magnitude by the scalar.



The resultant vector is the vector that is parallel to D_1 (opposite direction if the scalar is negative) and is 3 time as long.

Chap 3-4: Adding vectors by components

Example: A turtle goes 57 m due north, then to 30° south of east for 32 m. Find the resultant vector (that is, the final position).



$$V_x = V_{1x} + V_{2x} = 28 \text{ m}$$

$$V_y = V_{1y} + V_{2y} = 41 \text{ m}$$

$$V = \sqrt{V_x^2 + V_y^2} = 50 \text{ m}$$

$$\theta = \text{atan}(V_y / V_x) = 55.7^\circ$$

Chap 3-4: Adding vectors by components

STRATEGY:

Resolve each vector into components

$$V_{ix} = V_i \cos(\theta_i)$$

$$V_{iy} = V_i \sin(\theta_i)$$

Add the components by direction

$$V_x = V_{1x} + V_{2x} + \dots$$

$$V_y = V_{1y} + V_{2y} + \dots$$

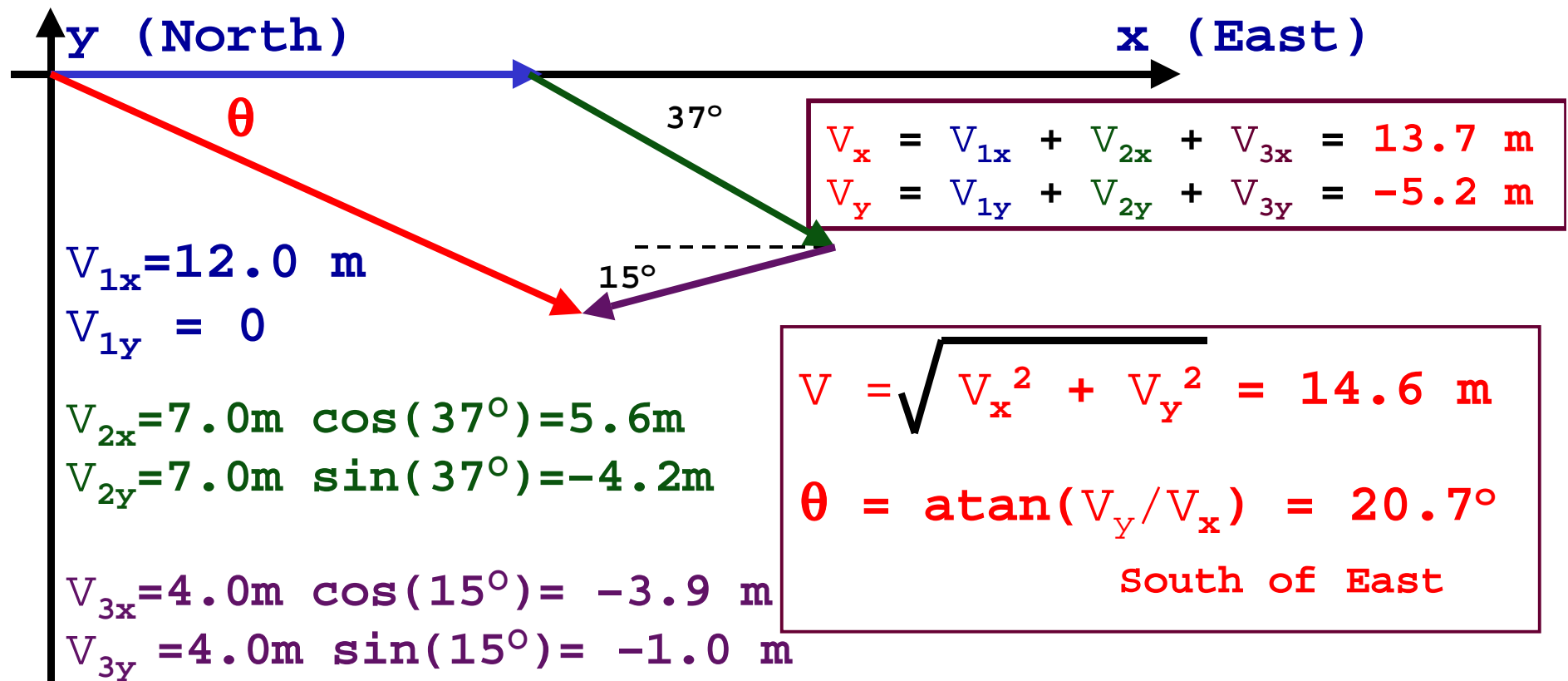
Find magnitude, direction of resultant vector

$$V = \sqrt{V_x^2 + V_y^2}$$

$$\theta = \text{atan}(V_y / V_x)$$

Chap 3-4: Adding vectors by components

Example: Two Drunken Sailors leave a bar. They walk 12.0 m due east, then 7.0 m 37° south of east, then 4.0 m 15° south of west. Where did they end their journey (fell down in drunken stupor, stopped by the cops, got into a fight, ...)?





The Whirlpool Galaxy is a classic spiral galaxy. At only 23 million light years distant and fully 65 thousand light years across, M51, also known as NGC 5194, is one of the brightest and most picturesque galaxies on the sky. The smaller galaxy appearing here below and to the left is well behind M51, as can be inferred by the dust in a foreground spiral arm blocking light from this smaller galaxy. The Whirlpool, pictured above, is visible with binoculars in the constellation of Canes Venaciti. M51 is a spiral galaxy of type Sc and is the dominant member of a whole group of galaxies. Astronomers speculate that M51's spiral structure is primarily due to its gravitational interaction with this smaller galaxy.