Chap 7: Linear Momentum

Other than energy there are other quantities that are conserved, Electric charge, and linear and angular momentum.

Chap 7–1: Momentum and Force

Linear momentum is defined to be the mass time the velocity

$$\mathbf{p} = m\mathbf{v}$$

v is a vector and so is p, p is in the same direction of v with a magnitude

$$p = mv$$

The SI unit for momentum is mv which is kg·m/s

The heavier and faster an object, the higher its momentum.

Chap 7–1: Momentum and Force

To change the momentum of an object, a force has to be applied.

Newton's 2nd Law of Motion, revisited:

The rate of change of the momentum of an object is equal to the net force applied to it.

$$\sum F = \frac{\Delta p}{\Delta t} = \frac{mv_2 - mv_1}{\Delta t} = \frac{m(v_2 - v_1)}{\Delta t} = m\frac{\Delta v}{\Delta t}$$

$$\sum F = ma$$

These are equivalent as long as the mass remains constant.

Chap 7–1: Momentum and Force

Example: Find the force exerted by water coming out of a hose at 13 m/s at a rate of 3 kg/s? Assume horizontal flow. No splashing.

At each second

$p_x = mv_x$ = **(3 kg)(13 m/s) = 39 kg·m/s**

When the water hits a wall it is stopped and its velocity is zero.

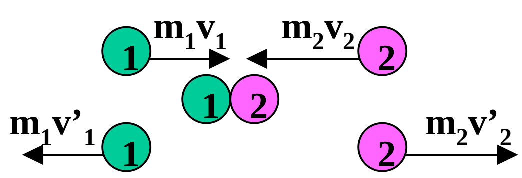
The force it takes to do that is

$$F = \frac{\Delta p}{\Delta t} = \frac{p_f - p_i}{\Delta t} = \frac{0 - 39\frac{kg \cdot m}{s}}{1s} = -39N$$

Chap 7–2: Conservation of Momentum

Two balls colliding, the momentum of each changes, the sum is conserved.

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2$$



Assume that the force that one ball exerts on the other during the collision is constant.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} \Rightarrow \Delta\vec{p} = \vec{F}\Delta t$$

Newton's 2nd

$$\Delta\vec{p}_2 = m_2\vec{v}_2 - m_2\vec{v}'_2 = \vec{F}_{21}\Delta t$$

$$\Delta\vec{p}_1 = m_1\vec{v}_1 - m_1\vec{v}'_1 = \vec{F}_{12}\Delta t = -\vec{F}_{21}\Delta t$$

Newton's 3rd

$$m_1\vec{v}_1 - m_1\vec{v}'_1 = -\left(m_2\vec{v}_2 - m_2\vec{v}'_2\right)$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}'_1 + m_2\vec{v}'_2$$

Conservation of momentum

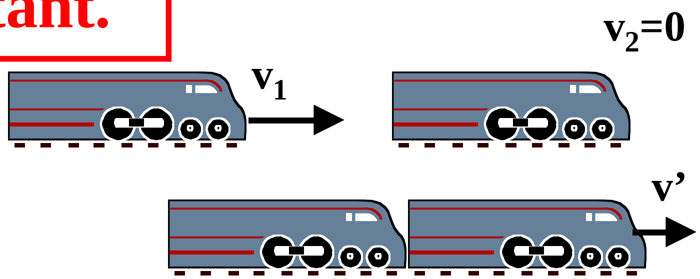
Chap 7–2: Conservation of Momentum

Momentum is conserved for any number of interacting bodies.

Law of conservation of momentum:

The total momentum of an isolated system of bodies remains constant.

Example: A 8,000 kg engine goes at 20 m/s and strikes a stationary engine. Find the final velocity v′.



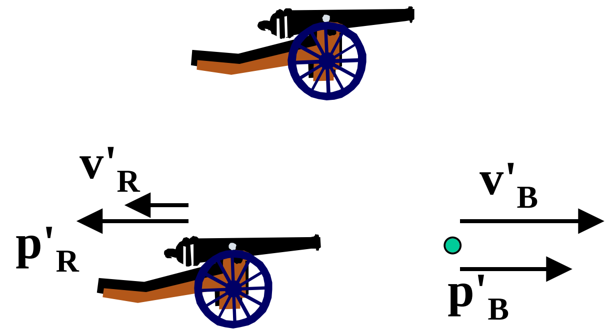
$$p_i = m_1 v_1 + m_2 v_2 = (m_1 + m_2) v' = p_f$$

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(8{,}000kg)\left(20\frac{m}{s}\right) + 0}{16{,}000kg} = 10\frac{m}{s}$$

Half the initial speed

Chap 7–2: Conservation of Momentum

Example: Find the recoil velocity of a 155 kg gun that shoots a 0.50 kg bullet at 100 m/s.



$$p_i = 0 = m_R v'_R + m_B v'_B = p_f$$

$$m_R v'_R = -m_B v'_B \Rightarrow v'_R = -\frac{m_B v'}{m_R}$$

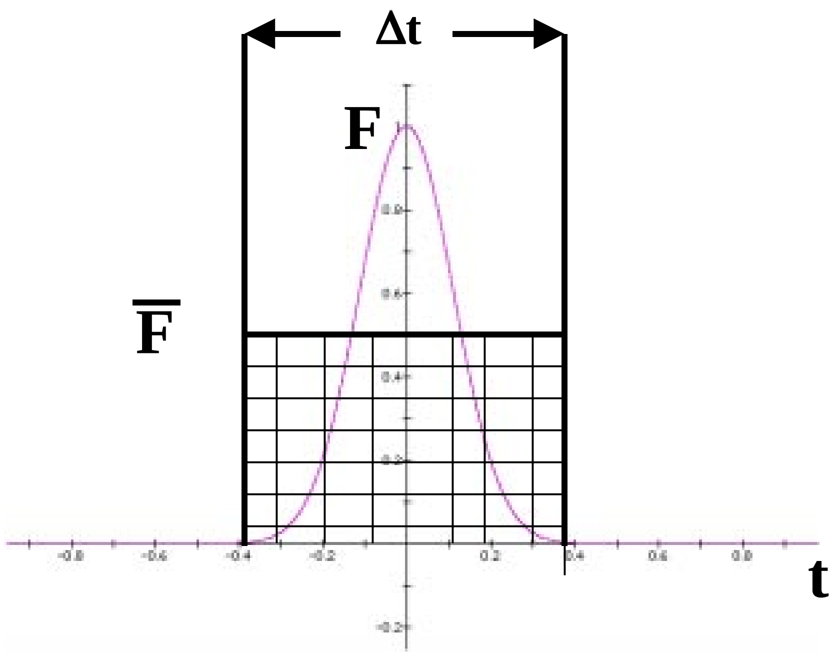
$$v'_R = -\frac{(0.50kg)\left(100\frac{m}{s}\right)}{155kg} = -0.32\frac{m}{s}$$

Chap 7–3: Collisions and Impulse

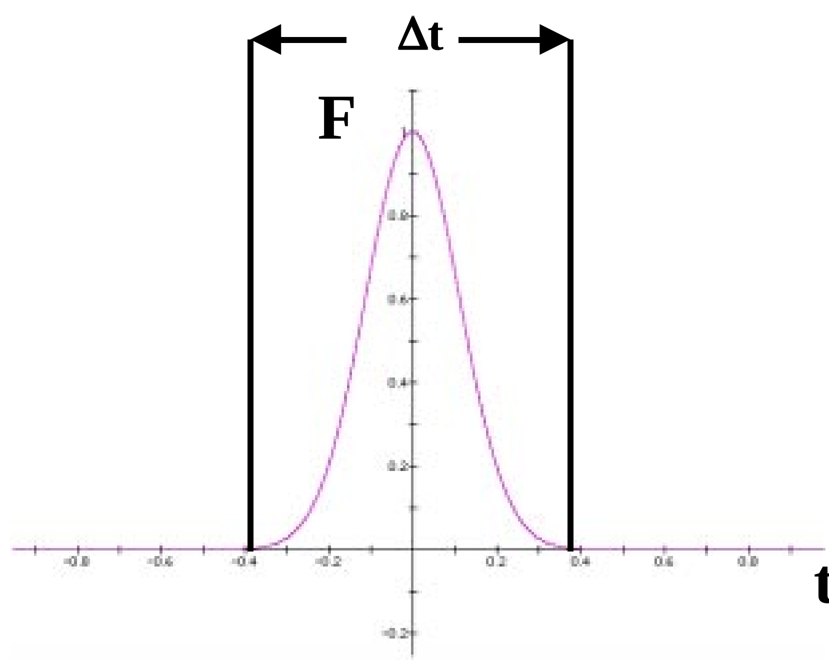
During collisions a large force is acting for a short time, during which the objects deform.

Newton's 2nd Law says $F = \Delta p / \Delta t$

Impulse is defined to be



Impulse = $F \, \Delta t = \Delta p$



Chap 7–3: Collisions and Impulse

Calculate the impulse experienced when a 50 kg person lands on firm ground after jumping from a height of 2.0 m.

The impulse = F Δt = Δp. Although we don't know the force we can find the change of momentum:

Conservation of energy: $\Delta KE = -\Delta PE$

$$\tfrac{1}{2}mv^2 - 0 = -mg(y - y_0)$$

$$v = \sqrt{2g(y-y_0)} = \sqrt{2\left(9.80\frac{m}{s^2}\right)(2.0m)} = 6.3\frac{m}{s}$$

The change in momentum is

$$\overline{F}\Delta t = \Delta p = m(v - v_0) = (50kg)\left(0 - 6.3\frac{m}{s}\right) = -313N \cdot s$$

Chap 7–3: Collisions and Impulse

Calculate the impulse experienced when a 50 kg person lands on firm ground after jumping from a height of 2.0 m.

Find the average force if the legs bent 0.025 m.

$$\bar{v} = \frac{d}{\Delta t} = \frac{v_f + v_i}{2} = \frac{0 + 6.3\frac{m}{s}}{2} = 3.1\frac{m}{s}$$

$$\Delta t = \frac{d}{\bar{v}} = \frac{0.025m}{3.1\frac{m}{s}} = 8.1 \cdot 10^{-3} s$$

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{313 N \cdot s}{8.1 \cdot 10^{-3} s} = 3.9 \cdot 10^{4} N$$

This is the total force, if we subtract the force of gravity, that is the weight that is always there we get:

$\bar{F} = F_{grd} - mg \Rightarrow F_{grd} = \bar{F} + mg$

$F_{grd} = 3.9 \cdot 10^4 N + (50kg)(9.80 m/s^2)$

$F_{grd} = 3.9 \cdot 10^4 N$

Chap 7–3: Collisions and Impulse

Calculate the impulse experienced when a 50 kg person lands on firm ground after jumping from a height of 2.0 m.

Find the average force if the legs bent 0.50 m.

$$\bar{v} = \frac{d}{\Delta t} = \frac{v_f + v_i}{2} = \frac{0 + 6.3\frac{m}{s}}{2} = 3.1\frac{m}{s}$$

$$\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.1\frac{m}{s}} = 0.16s$$

$$\bar{F} = \frac{\Delta p}{\Delta t} = \frac{313N \cdot s}{0.16s} = 1.9 \cdot 10^3 N$$

This is the total force, if we subtract the force of gravity, that is the weight that is always there we get:

$\overline{F} = F_{grd} - mg \Rightarrow F_{grd} = \overline{F} + mg$

$F_{grd} = 1.9 \cdot 10^3 N + (50kg)(9.80m/s^2)$

$F_{grd} = 2.4 \cdot 10^3 N$

Chap 7–4: Conservation of Energy and Momentum

In collisions we rarely know how the force varies with time. For collisions of hard objects with no heat production (billiard balls) both kinetic energy and momentum are conserved.

Let 1 and 2 denote the objects then:

$$^1/_2 m v_1^2 + {}^1/_2 m v_2^2 = {}^1/_2 m v'^2_1 + {}^1/_2 m v'^2_2$$

This situation is called elastic collision where the colliding objects

- **Do not stick to each other.**
- **Do not produce any heat.**
- **The collision is very short in duration.**

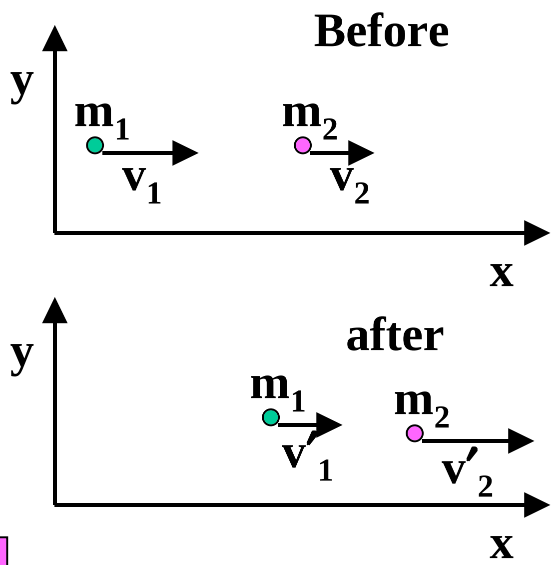
Chap 7–5: Elastic Collisions in One Dimension

Two particles move with velocities v_1 and v_2 and collide:

Conservation of Momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 (v_1 - v_1') = m_2 (v_2 - v_2') \qquad \textbf{(1)}$$



Conservation of Energy

$$\frac{1}{2} m v_1^{\,2} + \frac{1}{2} m v_2^{\,2} = \frac{1}{2} m v_1'^{\,2} + \frac{1}{2} m v_2'^{\,2}$$

$$m_1 (v_1^{\,2} - v_1'^{\,2}) = m_2 (v_2^{\,2} - v_2'^{\,2})$$

$$m_1 (v_1 + v_1')(v_1 - v_1') = m_2 (v_2 + v_2')(v_2 - v_2') \qquad \textbf{(2)}$$

Dividing (2) by (1)

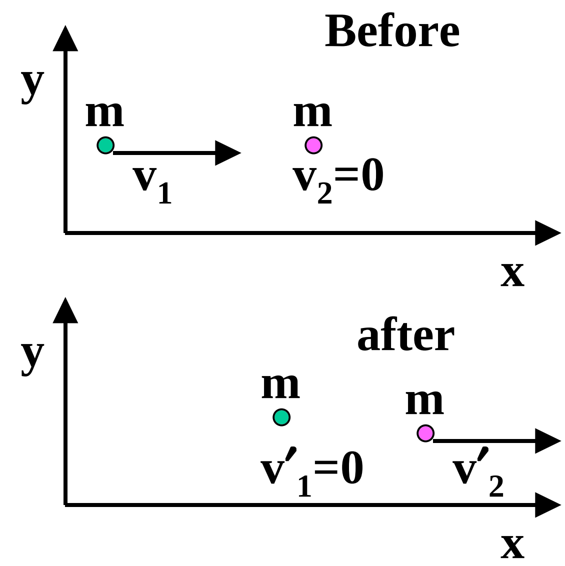
$$v_1 + v_1' = v_2 + v_2' \quad \Longrightarrow \quad v_1 - v_2 = -(v_1' - v_2')$$

Conservation of Energy for Elastic Collisions

$$v_1 - v_2 = -(v_1' - v_2')$$

Chap 7–5: Elastic Collisions in One Dimension

Example: A billiard ball moves with velocity v_1=v, hits a second ball of the same mass at rest, find the final velocities.



Conservation of momentum:

$$mv = mv_1' + mv_2' \Rightarrow v = v_1' + v_2'$$

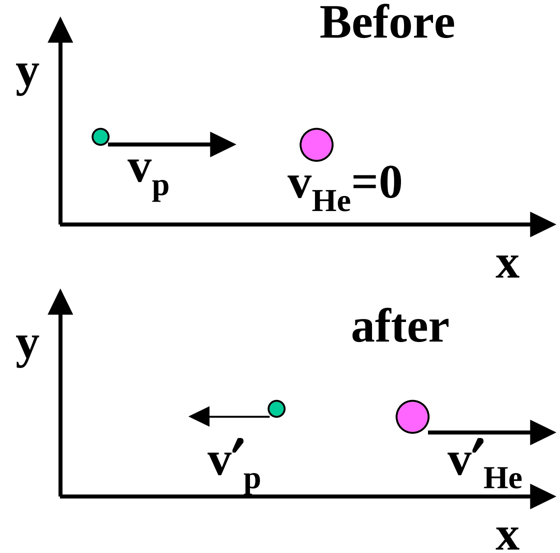
Conservation of energy:

$$v_1 - v_2 = v_2' - v_1' \Rightarrow v = v_2' - v_1'$$

$$v_1' = 0 \qquad \text{and} \qquad v_2' = v$$

Chap 7–5: Elastic Collisions in One Dimension

Example: A proton (m_p= 1.01 u) going at speed $3.60\cdot10^4$m/s hits a stationary helium nucleus (m_{He}=4.00u), find the final velocities.



Conservation of momentum:

$$m_p v_p + 0 = m_p v'_p + m_{He} v'_{He}$$

Conservation of energy:

$$v_p - 0 = v'_{He} - v'_p \Rightarrow v'_p = v'_{He} - v_p$$

$$m_p v_p = m_p v'_{He} - m_p v_p + m_{He} v'_{He}$$

$$v'_{He} = \frac{2m_p v_p}{m_p + m_{He}} = \frac{2(1.01u)\left(3.60\cdot10^4\frac{m}{s}\right)}{1.01u + 4.00u} = 1.45\cdot10^4\frac{m}{s}$$

$$v'_p = v'_{He} - v_p = 1.45\cdot10^4\frac{m}{s} - 3.60\cdot10^4\frac{m}{s} = -2.15\cdot10^4\frac{m}{s}$$

Extrasolar Planet Candidates Around Sun–like Stars